

# Scanner

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\* Course website: <https://www.cs.columbia.edu/~rgu/courses/4115/spring2019>

\*\* These slides are borrowed from Prof. Edwards.

# The Big Picture

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# The First Question

How do we describe/construct a program?

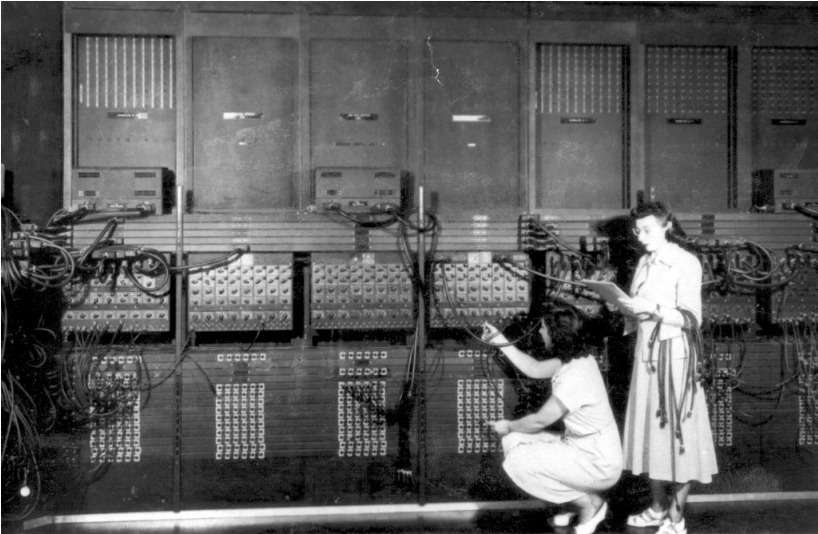
## Use continuously varying values?



Very efficient, but has serious noise issues

Edison Model B Home Cylinder phonograph, 1906

# The ENIAC: Programming with Spaghetti



## Have one symbol per program?



Works nicely when there are only a few things

Sholes and Glidden Typewriter, E. Remington and Sons, 1874

# Have one symbol per program?

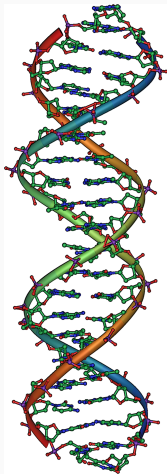


Not so good when there are many, many things

Nippon Typewriter SH-280, 2268 keys

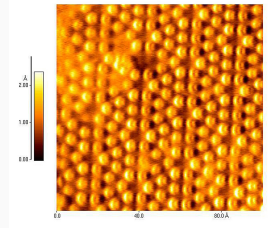
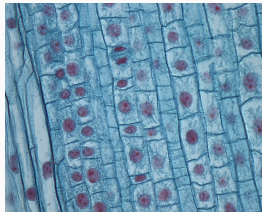
# Solution: Use a Discrete Combinatorial System

Use combinations of a small number of things to represent (exponentially) many different things.



ENGLISH SOUNDS

(1)	(2)	(3)	(4)	(5)	(6)		
chinese	FISH	WOMAN	BOOK	TOOTH	RADIO		
(7)	(8)	(9)	(10)	(11)	(12)	(13)	
elephant	CAMERA	BIRD	BALL	CAKE	BUG	PHONE	
(14)	(15)	(16)	(17)	(18)	(19)	(20)	
FAT	HAT	CAR	LOCK	MILK	KNIFE	COW	
(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
POT	BOTTLE	TABLE	STAIR	CHOPSTICKS	LEAF	KEY	SPOUT
(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)
SPOWER	VAN	UMBRELLA	BEAVER	SNAKE	YOK	CHANDLER	ROCK
(37)	(38)	(39)	(40)	(41)	(42)	(43)	(44)
MOUSE	AIR CAPACITOR	WING	HOUSE	LIGHT	RING	WINK	YACHT

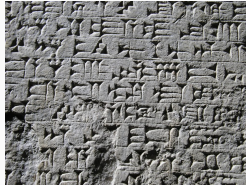




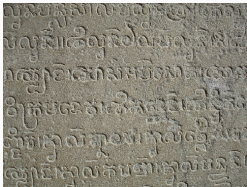
# Every Human Writing System Does This



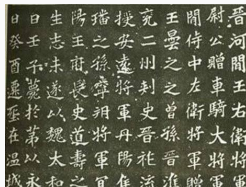
Hieroglyphics (24+)



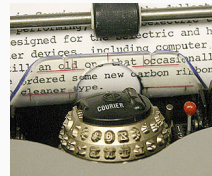
Cuneiform (1000 - 300)



Sanskrit (36)



Chinese (214 - 4000)



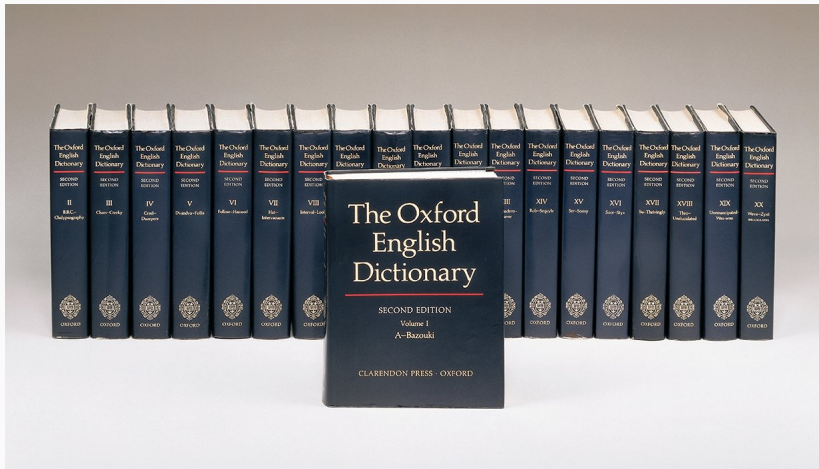
IBM Selectric (88-96)



## The Second Question

How do we describe the combinations of a small number of things.

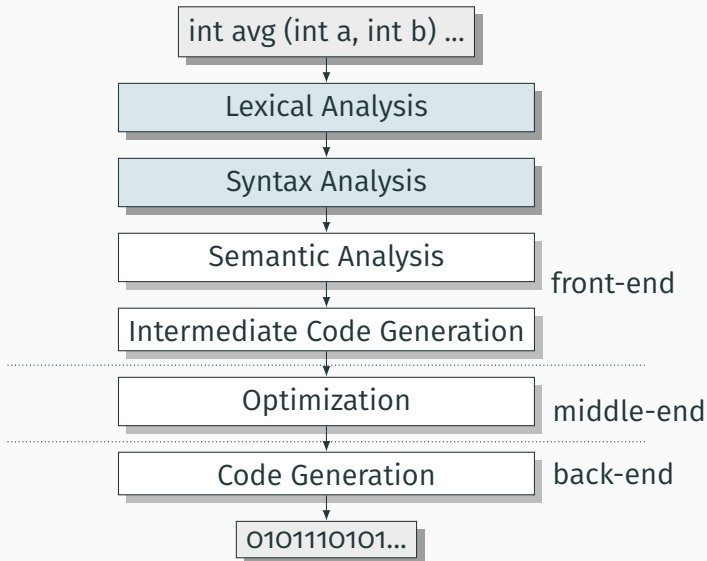
# Just List Them?



Gets annoying for large numbers of combinations



# Scanning and Parsing



# Lexical Analysis

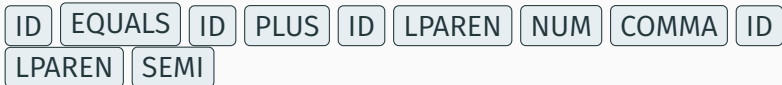
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# Lexical Analysis (Scanning)

Translate a stream of characters to a stream of tokens



f o o \_ = \_ a + \_ bar ( 0 , \_ 42 , \_ q ) ;



Token	Lexemes	Pattern
EQUALS	=	an equals sign
PLUS	+	a plus sign
ID	a foo bar	letter followed by letters or digits
NUM	0 42	one or more digits

# Lexical Analysis

Goal: simplify the job of the parser and reject some wrong programs, e.g.,

```
%#$ ^ #!##%#$
```

is not a C program<sup>†</sup>

Scanners are usually much faster than parsers.

Discard as many irrelevant details as possible (e.g., whitespace, comments).

Parser does not care that the identifier is “supercalifragilisticexpialidocious.”

Parser rules are only concerned with tokens.

<sup>†</sup> It is what you type when your head hits the keyboard



# Describing Tokens

**Alphabet:** A finite set of symbols

Examples:  $\{ 0, 1 \}$ ,  $\{ A, B, C, \dots, Z \}$ , ASCII, Unicode

**String:** A finite sequence of symbols from an alphabet

Examples:  $\epsilon$  (the empty string), Ronghui,  $\alpha\beta\gamma$

**Language:** A set of strings over an alphabet

Examples:  $\emptyset$  (the empty language),  $\{ 1, 11, 111, 1111 \}$ , all English words, strings that start with a letter followed by any sequence of letters and digits

# Operations on Languages

Let  $L = \{ \epsilon, \text{wo} \}$ ,  $M = \{ \text{man, men} \}$

**Concatenation:** Strings from one followed by the other

$LM = \{ \text{man, men, woman, women} \}$

**Union:** All strings from each language

$L \cup M = \{ \epsilon, \text{wo, man, men} \}$

**Kleene Closure:** Zero or more concatenations

$M^* = \{ \epsilon \} \cup M \cup MM \cup MMM \dots =$   
 $\{ \epsilon, \text{man, men, manman, manmen, menman, menmen,}$   
 $\text{manmanman, manmanmen, manmenman, ...} \}$

## Regular Expressions over an Alphabet $\Sigma$

A standard way to express languages for tokens.

1.  $\epsilon$  is a regular expression that denotes  $\{\epsilon\}$
2. If  $a \in \Sigma$ ,  $a$  is an RE that denotes  $\{a\}$
3. If  $r$  and  $s$  denote languages  $L(r)$  and  $L(s)$ ,

$(r) \mid (s)$  denotes  $L(r) \cup L(s)$

$(r)(s)$   $\{tu : t \in L(r), u \in L(s)\}$

$(r)^*$   $\cup_{i=0}^{\infty} L(r)^i$

where  $L(r)^0 = \{\epsilon\}$

and  $L(r)^i = L(r)L(r)^{i-1}$

# Regular Expression Examples

$$\Sigma = \{a, b\}$$

Regexp.	Language
$a \mid b$	$\{a, b\}$
$(a \mid b)(a \mid b)$	$\{aa, ab, ba, bb\}$
$a^*$	$\{\epsilon, a, aa, aaa, aaaa, \dots\}$
$(a \mid b)^*$	$\{\epsilon, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, \dots\}$
$a \mid a^*b$	$\{a, b, ab, aab, aaab, aaaab, \dots\}$

## Specifying Tokens with REs

ID: letter followed by letters or digits

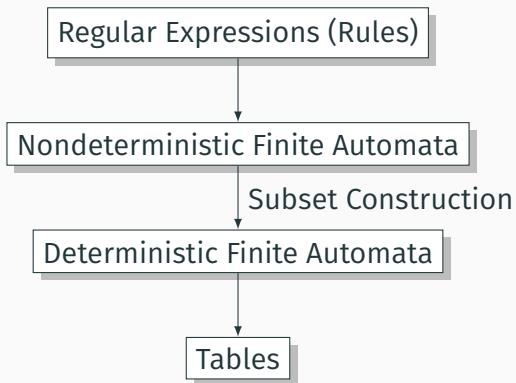
Typical choice:  $\Sigma = \text{ASCII characters, i.e.,}$   
 $\{\_, !, ", \#, \$, \dots, 0, 1, \dots, 9, \dots, A, \dots, Z, \dots, \sim\}$

**letters:**  $A \mid B \mid \dots \mid Z \mid a \mid \dots \mid z$

**digits:**  $0 \mid 1 \mid \dots \mid 9$

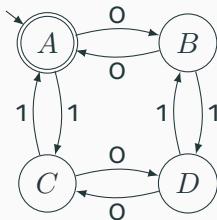
**identifier:**  $\text{letter} (\text{letter} \mid \text{digit})^*$

# Implementing Scanners Automatically



# Nondeterministic Finite Automata

“All strings containing an even number of 0’s and 1’s”



1. Set of states

$$S : \left\{ \textcircled{\textcircled{A}} \textcircled{B} \textcircled{C} \textcircled{D} \right\}$$

2. Set of input symbols  $\Sigma : \{0, 1\}$

3. Transition function

$$\sigma : S \times \Sigma_{\epsilon} \rightarrow 2^S$$

state	$\epsilon$	0	1
A	$\emptyset$	{B}	{C}
B	$\emptyset$	{A}	{D}
C	$\emptyset$	{D}	{A}
D	$\emptyset$	{C}	{B}

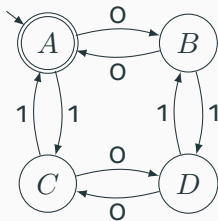
4. Start state  $s_0 : \textcircled{\textcircled{A}}$

5. Set of accepting states

$$F : \left\{ \textcircled{\textcircled{A}} \right\}$$

## The Language induced by an NFA

An NFA accepts an input string  $x$  iff there is a path from the start state to an accepting state that “spells out”  $x$ .



Show that the string “010010” is accepted.





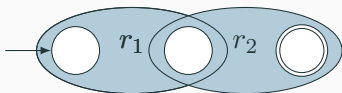
# Translating REs into NFAs (Thompson's algorithm)

$a$



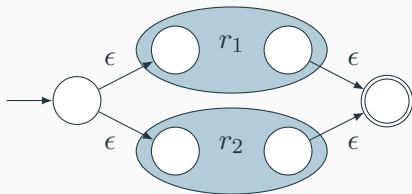
Symbol

$r_1 r_2$



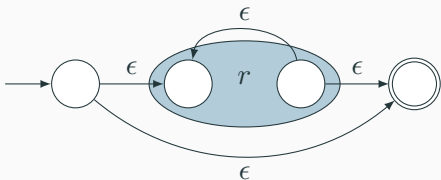
Sequence

$r_1 \mid r_2$



Choice

$(r)^*$

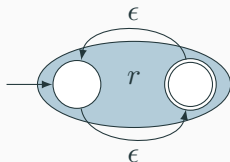


Kleene Closure

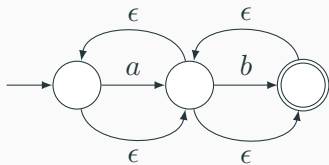
## Why So Many Extra States and Transitions?

Invariant: Single start state; single end state; at most two outgoing arcs from any state: helpful for simulation.

What if we used this simpler rule for Kleene Closure?



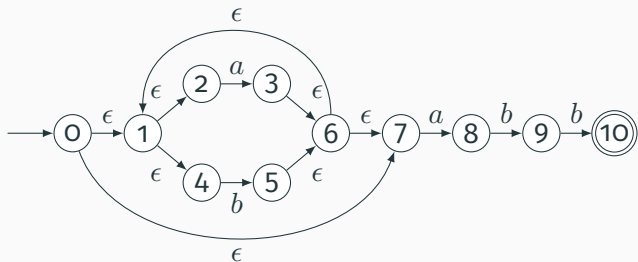
Now consider  $a^*b^*$  with this rule:



Is this right?

# Translating REs into NFAs

Example: Translate  $(a | b)^*abb$  into an NFA. Answer:



Show that the string "aabb" is accepted. Answer:



# Simulating NFAs

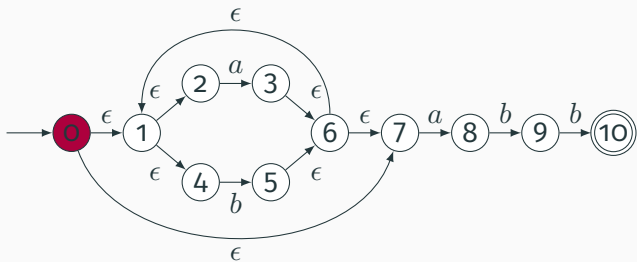
Problem: you must follow the “right” arcs to show that a string is accepted. How do you know which arc is right?

Solution: follow them all and sort it out later.

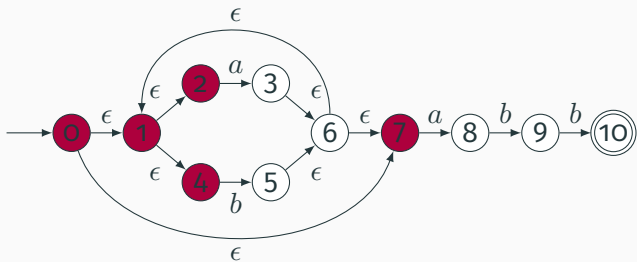
“Two-stack” NFA simulation algorithm:

1. Initial states: the  $\epsilon$ -closure of the start state
2. For each character  $c$ ,
  - New states: follow all transitions labeled  $c$
  - Form the  $\epsilon$ -closure of the current states
3. Accept if any final state is accepting

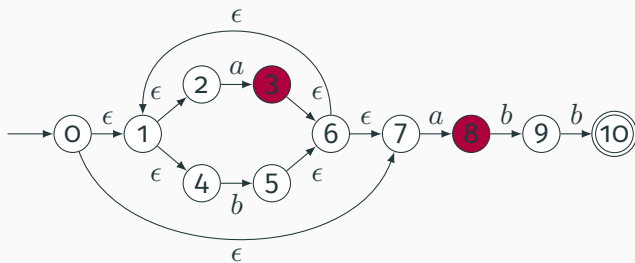
## Simulating an NFA: $\cdot aabb$ , Start



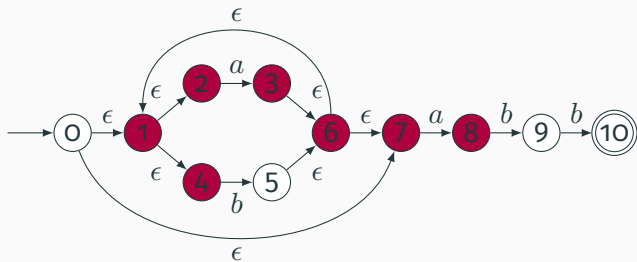
## Simulating an NFA: $\cdot aabb$ , $\epsilon$ -closure



## Simulating an NFA: $a \cdot abb$

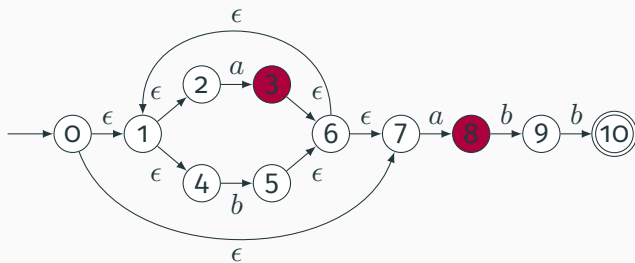


## Simulating an NFA: $a \cdot abb$ , $\epsilon$ -closure

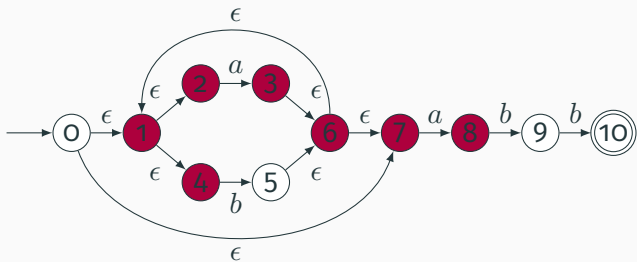




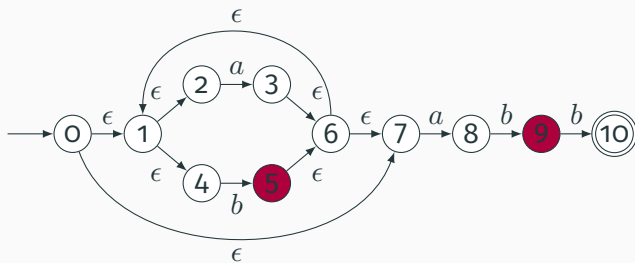
# Simulating an NFA: $aa \cdot bb$



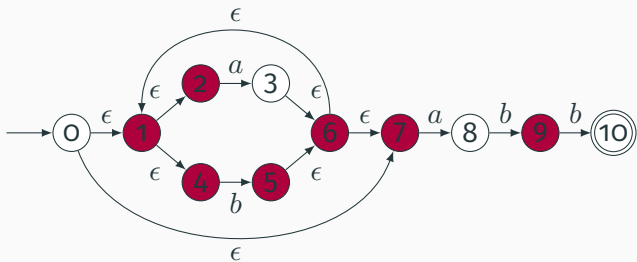
## Simulating an NFA: $aa \cdot bb$ , $\epsilon$ -closure



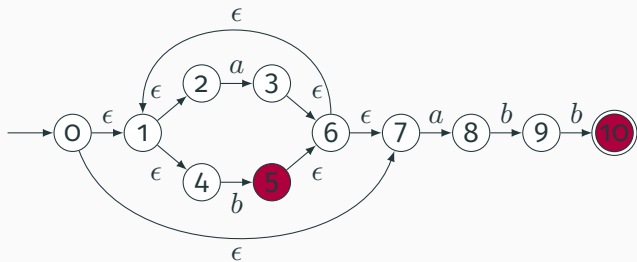
## Simulating an NFA: $aab \cdot b$



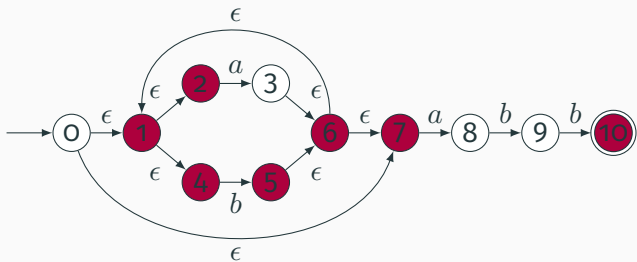
## Simulating an NFA: $aab \cdot b$ , $\epsilon$ -closure



## Simulating an NFA: $aabb$ .



## Simulating an NFA: $aabb\cdot$ , Done



# Deterministic Finite Automata

Restricted form of NFAs:

- No state has a transition on  $\epsilon$
- For each state  $s$  and symbol  $a$ , there is at most one edge labeled  $a$  leaving  $s$ .

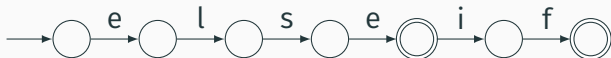
Differs subtly from the definition used in COMS W3261 (Sipser, *Introduction to the Theory of Computation*)

Very easy to check acceptance: simulate by maintaining current state. Accept if you end up on an accepting state. Reject if you end on a non-accepting state or if there is no transition from the current state for the next symbol.

# Deterministic Finite Automata

```
{  
  type token = ELSE | ELSEIF  
}
```

```
rule token =  
  parse "else" { ELSE }  
  | "elseif" { ELSEIF }
```





# Deterministic Finite Automata

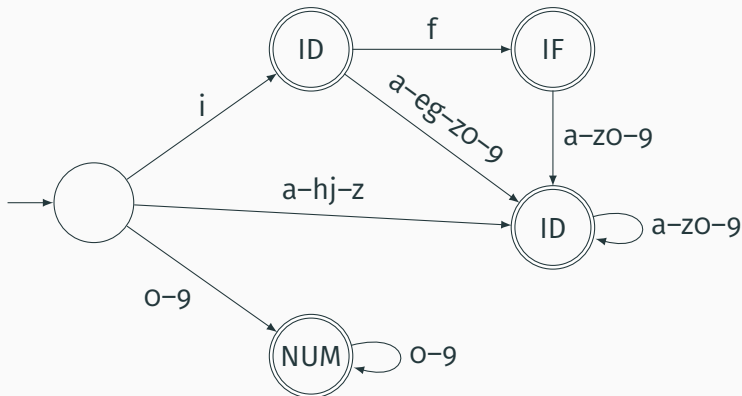
```
{ type token = IF | ID of string | NUM of string }
```

```
rule token =
```

```
  parse "if"
```

```
    | [ 'a' - 'z' ] [ 'a' - 'z' '0' - '9' ]* as lit { IF }
```

```
    | [ '0' - '9' ]+ as num { NUM(num) }
```



## Building a DFA from an NFA

Subset construction algorithm

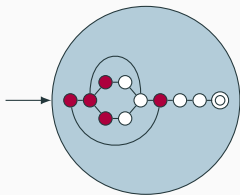
Simulate the NFA for all possible inputs and track the states that appear.

Each unique state during simulation becomes a state in the DFA.

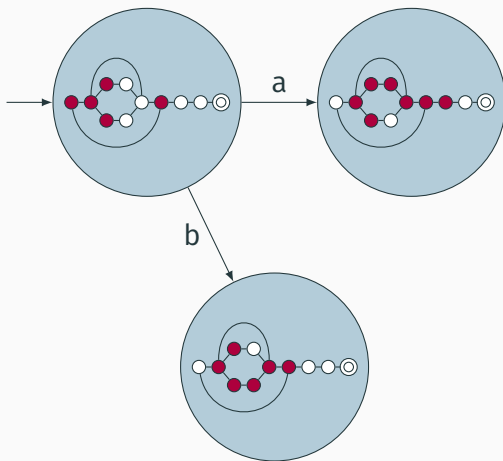
# The Subset Construction Algorithm

1. Create the start state of the DFA by taking the  $\epsilon$ -closure of the start state of the NFA.
2. Perform the following for the new DFA state: For each possible input symbol:
  - Apply move to the newly-created state and the input symbol; this will return a set of states.
  - Apply the  $\epsilon$ -closure to this set of states, possibly resulting in a new set. This set of NFA states will be a single state in the DFA.
3. Each time we generate a new DFA state, we must apply step 2 to it. The process is complete when applying step 2 does not yield any new states.
4. The finish states of the DFA are those which contain any of the finish states of the NFA.

# Subset construction for $(a | b)^*abb$

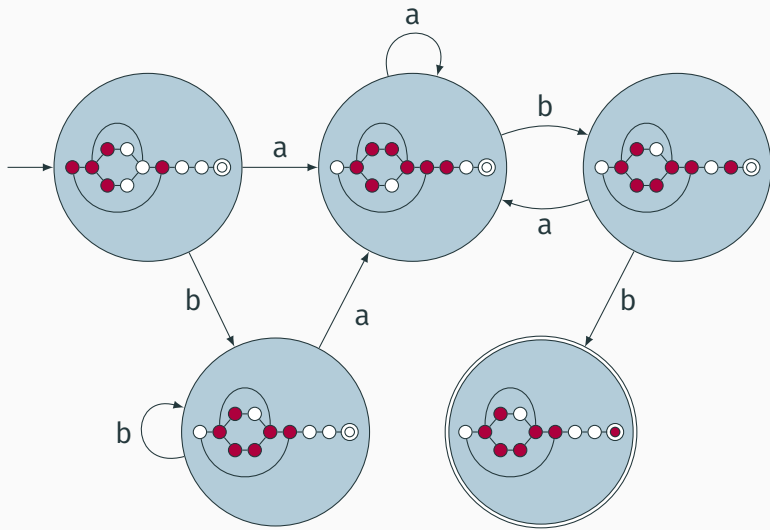


# Subset construction for $(a | b)^*abb$

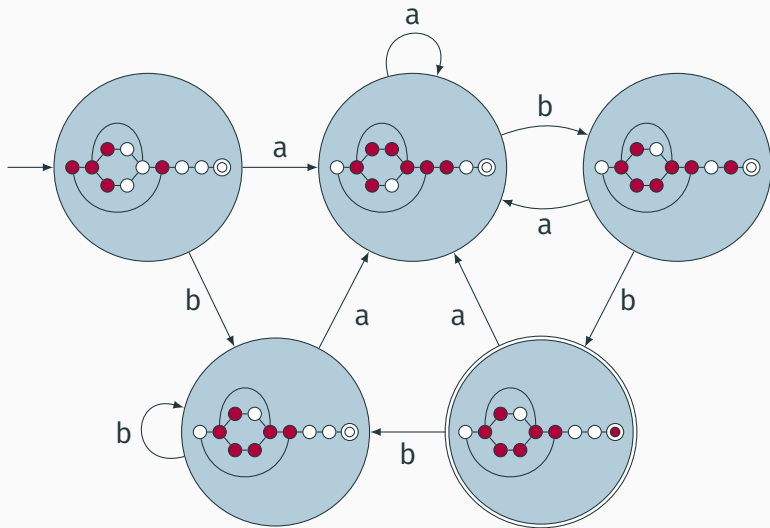




# Subset construction for $(a \mid b)^*abb$

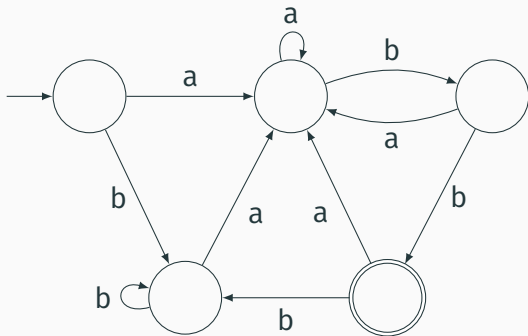


# Subset construction for $(a \mid b)^*abb$



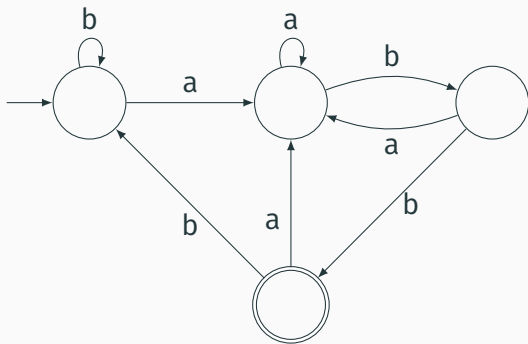


## Result of subset construction for $(a | b)^*abb$



*Is this minimal?*

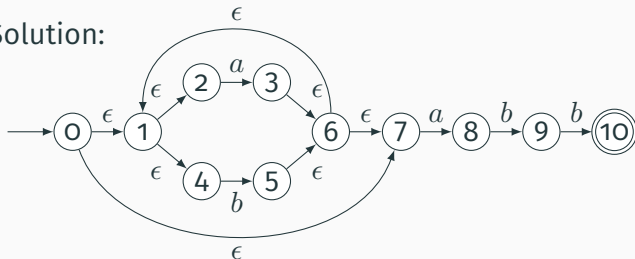
# Minimized result for $(a | b)^*abb$



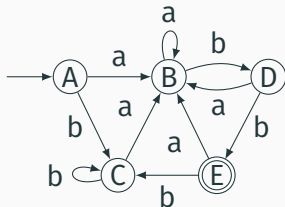
# Transition Table Used In the Dragon Book

Problem: Translate  $(a | b)^*abb$  into an NFA and perform subset construction to produce a DFA.

Solution:



NFA State	DFA State	a	b
{0,1,2,4,7}	A	B	C
{1,2,3,4,6,7,8}	B	B	D
{1,2,4,5,6,7}	C	B	C
{1,2,4,5,6,7,9}	D	B	E
{1,2,4,5,6,7,10}	E	B	C



## Subset Construction

An DFA can be exponentially larger than the corresponding NFA.

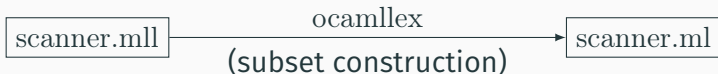
$n$  states versus  $2^n$

Tools often try to strike a balance between the two representations.

# Lexical Analysis with Ocamllex

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# Constructing Scanners with Ocamllex



An example:

scanner.mll

```
{ open Parser }  
  
rule token =  
  parse [ ' ' '\t' '\r' '\n' ] { token lexbuf }  
  | '+' { PLUS }  
  | '-' { MINUS }  
  | '*' { TIMES }  
  | '/' { DIVIDE }  
  | ['0'-'9']+ as lit { LITERAL(int_of_string lit) }  
  | eof { EOF }
```

# Ocamlex Specifications

```
{
  (* Header: verbatim OCaml code; mandatory *)
}

(* Definitions: optional *)
let ident = regexp
let ...

(* Rules: mandatory *)
rule entrypoint1 [arg1 ... argn] =
  parse pattern1 { action (* OCaml code *) }
  | ...
  | patternn { action }
and entrypoint2 [arg1 ... argn]} =
  ...
and ...

{
  (* Trailer: verbatim OCaml code; optional *)
}
```

## Patterns (In Order of Decreasing Precedence)

Pattern	Meaning
'c'	A single character
<u>—</u>	Any character (underline)
eof	The end-of-file
"foo"	A literal string
[ '1' '5' 'a'-'z' ]	"1," "5," or any lowercase letter
[ ^ '0'-'9' ]	Any character except a digit
( <i>pattern</i> )	Grouping
<i>identifier</i>	A pattern defined in the let section
<i>pattern</i> *	Zero or more <i>patterns</i>
<i>pattern</i> +	One or more <i>patterns</i>
<i>pattern</i> ?	Zero or one <i>patterns</i>
<i>pattern</i> <sub>1</sub> <i>pattern</i> <sub>2</sub>	<i>pattern</i> <sub>1</sub> followed by <i>pattern</i> <sub>2</sub>
<i>pattern</i> <sub>1</sub>   <i>pattern</i> <sub>2</sub>	Either <i>pattern</i> <sub>1</sub> or <i>pattern</i> <sub>2</sub>
<i>pattern</i> as <i>id</i>	Bind the matched pattern to variable <i>id</i>



## An Example

```
{ type token = PLUS | IF | ID of string | NUM of int }
let letter = ['a'-'z' 'A'-'Z']
let digit = ['0'-'9']

rule token =
  parse [' ' '\n' '\t'] { token lexbuf } (* Ignore whitespace *)
    | '+' { PLUS } (* A symbol *)
    | "if" { IF } (* A keyword *)
    | letter (letter | digit | '_' )* as id { ID(id) } (* Identifiers *)
    | digit+ as lit { NUM(int_of_string lit) } (* Numeric literals *)
    | "/*" { comment lexbuf } (* C-style comments *)

and comment =
  parse "*/" { token lexbuf } (* Return to normal scanning *)
    | _ { comment lexbuf } (* Ignore other characters *)
```

## Nested Comments

```
{ type token = PLUS | ID of string | NUM of int }

let letter = ['a'-'z' 'A'-'Z']
let digit = ['0'-'9']

rule token =
  parse [ ' ' '\n' '\t' ] { token lexbuf } (* Ignore whitespace *)

  | '+' { PLUS } (* A symbol *)

  | letter (letter | digit | '_')* as id { ID(id) }
  | digit+ as lit { NUM(int_of_string lit) }

  | "/*" { comment 0 lexbuf } (* C-style comment *)

and comment level =
  parse "*/" { if level == 0 then token lexbuf
               else comments (level - 1) lexbuf }
  | "/*" { comment (level + 1) lexbuf }
  | _ { comment level lexbuf } (* Ignore other characters *)
```

Typical style arising from scanner/parser division

Program text is a series of tokens possibly separated by whitespace and comments, which are both ignored.

- keywords (if while)
- punctuation (, ( +)
- identifiers (foo bar)
- numbers (10 -3.14159e+32)
- strings ("A String")

## Free-Format Languages

Java      C      C++      C#      Algol      Pascal

Some deviate a little (e.g., C and C++ have a separate preprocessor)

But not all languages are free-format.

The Python scripting language groups with indentation

```
i = 0
while i < 10:
    i = i + 1
    print i      # Prints 1, 2, ..., 10

i = 0
while i < 10:
    i = i + 1
print i        # Just prints 10
```

This is succinct, but can be error-prone.

How do you wrap a conditional around instructions?

## Syntax and Language Design

Does syntax matter? Yes and no

More important is a language's *semantics*—its meaning.

The syntax is aesthetic, but can be a religious issue.

But aesthetics matter to people, and can be critical.

Verbosity does matter: smaller is usually better.

Too small can be problematic: APL is a succinct language with its own character set.

There are no APL programs, only puzzles.

## Syntax and Language Design

Some syntax is error-prone. Classic FORTRAN example:

```
DO 5 I = 1,25 ! Loop header (for i = 1 to 25)
DO 5 I = 1.25 ! Assignment to variable DO5I
```

Trying too hard to reuse existing syntax in C++:

```
vector< vector<int> > foo;
vector<vector<int>> foo; // Syntax error
```

C distinguishes `>` and `>>` as different operators.

Bjarne Stroustrup tells me they have finally fixed this.