Scanner

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Columbia University

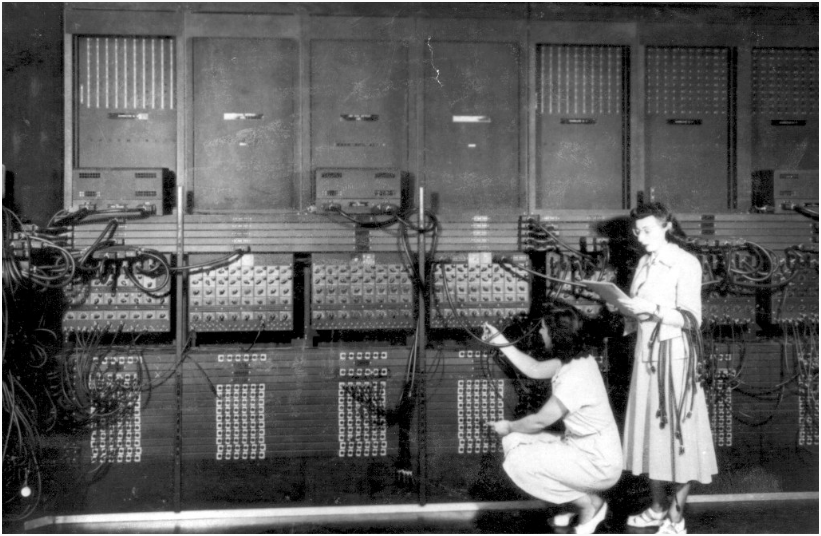
* Course website: https://verigu.github.io/4115Spring2024/

The Big Picture

The First Question

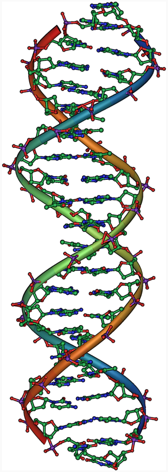
How do we describe/construct a program?

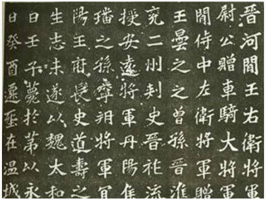
The ENIAC: Programming with Spaghetti

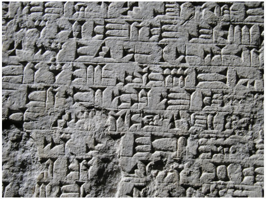


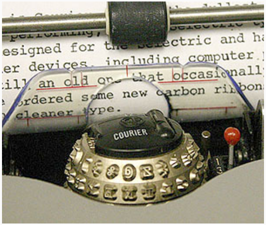
Solution: Use a Discrete Combinatorial System

Use combinations of a small number of things to represent (exponentially) many different things.





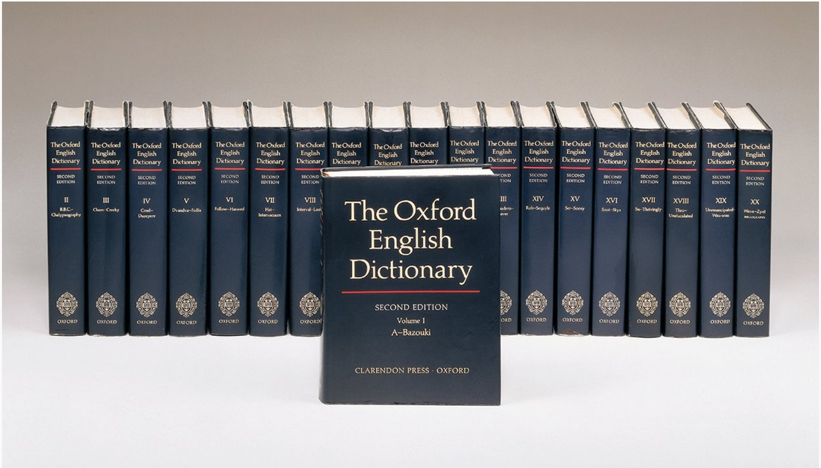




The Second Question

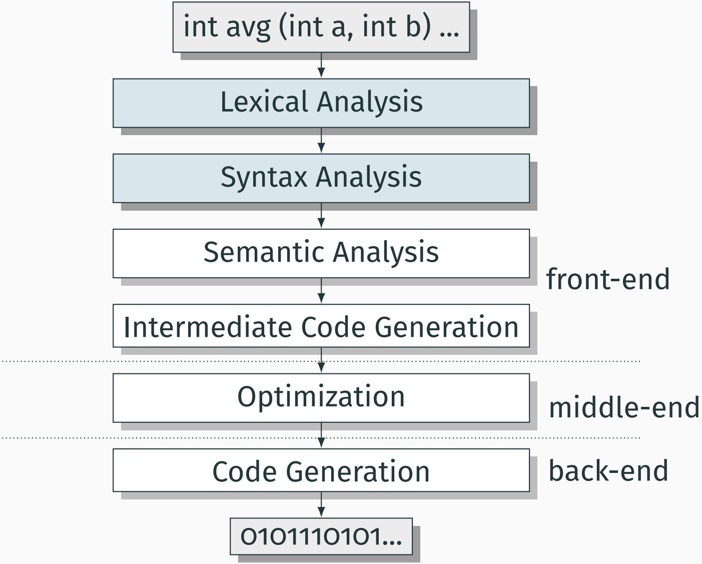
How do we describe the combinations of a small number of things.

Just List Them?



Gets annoying for large numbers of combinations

Scanning and Parsing



Lexical Analysis

Lexical Analysis (Scanning)

Translate a stream of characters to a stream of tokens

$\mathrm{f\ o\ o} \llcorner = \llcorner \mathrm{a} + \llcorner \mathrm{bar} (0 , \llcorner 42 , \llcorner \mathrm{q}) ;$

$\boxed{\text{ID}}$ $\boxed{\text{EQUALS}}$ $\boxed{\text{ID}}$ $\boxed{\text{PLUS}}$ $\boxed{\text{ID}}$ $\boxed{\text{LPAREN}}$ $\boxed{\text{NUM}}$ $\boxed{\text{COMMA}}$ $\boxed{\text{ID}}$ $\boxed{\text{LPAREN}}$ $\boxed{\text{SEMI}}$

Token	**Lexemes**	**Pattern**
EQUALS	$=$	an equals sign
PLUS	$+$	a plus sign
ID	a foo bar	letter followed by letters or digits
NUM	0 42	one or more digits

Lexical Analysis

Goal: simplify the job of the parser and reject some wrong programs, e.g.,

```
%#$^#!#%#$
```

is not a C program†

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```
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Discard as many irrelevant details as possible (e.g., whitespace, comments).

Parser does not care that the identifer is "supercalifragilisticexpialidocious."

Parser rules are only concerned with tokens.

[†] It is what you type when your head hits the keyboard

Describing Tokens

Alphabet: A finite set of symbols

Examples: $\{$ 0, 1 $\}$, $\{$ A, B, C, $\ldots$, Z $\}$, ASCII, Unicode

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Examples: ϵ (the empty string), Ronghui, $\alpha\beta\gamma$

Set: A set of strings over an alphabet

Examples: $\emptyset$ (the empty language), $\{$ 1, 11, 111, 1111 $\}$, all English words, strings that start with a letter followed by any sequence of letters and digits

Operations on String Sets

Let $L = \{ \epsilon, \text{wo} \}$, $M = \{ \text{man}, \text{men} \}$

Concatenation: Strings from one followed by the other

$LM = \{ \text{man}, \text{men}, \text{woman}, \text{women} \}$

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Union: All strings from each string set

$L \cup M = \{\epsilon,$ wo, man, men $\}$

Kleene Closure: Zero or more concatenations

$M^* = \{\epsilon\} \cup M \cup MM \cup MMM \cdots =$
$\{\epsilon,$ man, men, manman, manmen, menman, menmen, manmanman, manmanmen, manmenman, $\ldots\}$

Regular Expressions over an Alphabet Σ

A standard way to express tokens.

1. ϵ is a regular expression that denotes $\{\epsilon\}$
2. If $a \in \Sigma$, a is an RE that denotes $\{a\}$
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3. If r and s denote sets $L(r)$ and $L(s)$,

$(r) \mid (s)$	denotes	$L(r) \cup L(s)$
$(r)(s)$		$\{tu : t \in L(r), u \in L(s)\}$
$(r)^*$		$\cup_{i=0}^{\infty} L(r)^i$
	where	$L(r)^0 = \{\epsilon\}$
	and	$L(r)^i = L(r)L(r)^{i-1}$

Regular Expression Examples

$\Sigma = \{a, b\}$

Regexp.	String Set
$a \mid b$	$\{a, b\}$
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$a \mid b$	$\{a, b\}$
$(a \mid b)(a \mid b)$	$\{aa, ab, ba, bb\}$
$(a \mid b)^*$	$\{\epsilon, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, \ldots\}$
$a \mid a^*b$	$\{a, b, ab, aab, aaab, aaaab, \ldots\}$

Specifying Tokens with REs

ID: letter followed by letters or digits

Typical choice: $\Sigma =$ ASCII characters, i.e., $\{\text{␣}, !, ", \#, \$, \ldots, 0, 1, \ldots, 9, \ldots, A, \ldots, Z, \ldots, \tilde{}\}$

letters: $A \mid B \mid \cdots \mid Z \mid a \mid \cdots \mid z$

digits: $0 \mid 1 \mid \cdots \mid 9$

identifier:

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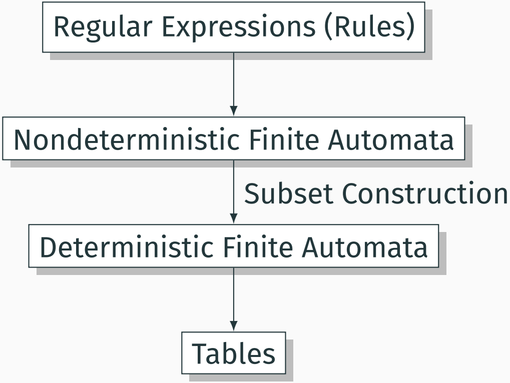
letters: $A \mid B \mid \cdots \mid Z \mid a \mid \cdots \mid z$

digits: $0 \mid 1 \mid \cdots \mid 9$

identifier: **letter** (**letter** | **digit**)*

Implementing Scanners Automatically



Finite Automata

“All strings containing an even number of 0’s and 1’s”

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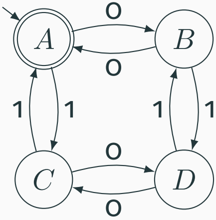

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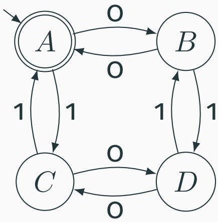


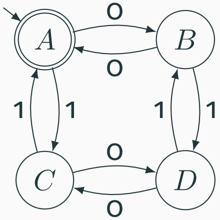
1. Set of states
   $S : \{ A \quad B \quad C \quad D \}$
2. Set of input symbols $\Sigma : \{0, 1\}$
3. Transition function
   $\sigma : S \times \Sigma_\epsilon \to 2^S$

state	ϵ	0	1
A	$\emptyset$	$\{B\}$	$\{C\}$
B	$\emptyset$	$\{A\}$	$\{D\}$
C	$\emptyset$	$\{D\}$	$\{A\}$
D	$\emptyset$	$\{C\}$	$\{B\}$

4. Start state s_0 : A
5. Set of accepting states
   $F : \{ A \}$

The String Set induced by an NFA

An NFA accepts an input string x iff there is a path from the start state to an accepting state that “spells out” x.



Show that the string 010010 is accepted.

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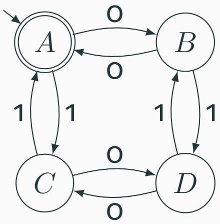

Show that the string 010010 is accepted.

$$A \xrightarrow{0} B \xrightarrow{1} D \xrightarrow{0} C \xrightarrow{0} D \xrightarrow{1} B \xrightarrow{0} A$$

Translating REs into NFAs (Thompson's algorithm)

a

Symbol

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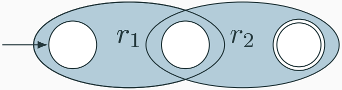
$r_1 r_2$



Sequence

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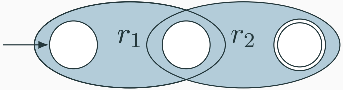

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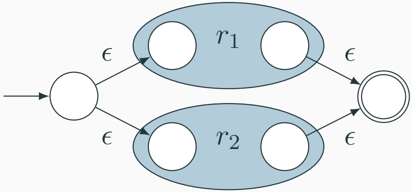

Choice

Translating REs into NFAs (Thompson’s algorithm)

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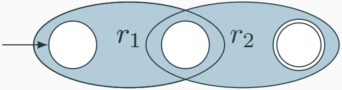

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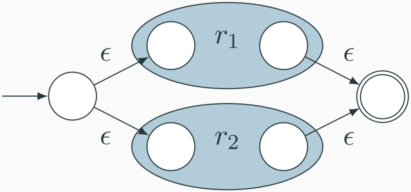
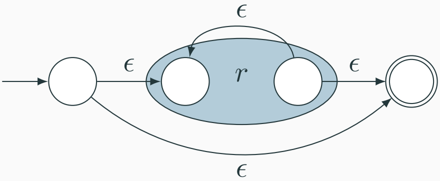


Choice

$(r)^*$

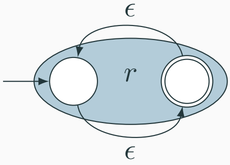


Kleene Closure

Why So Many Extra States and Transitions?

Invariant: Single start state; single end state; at most two outgoing arcs from any state: helpful for simulation.

What if we used this simpler rule for Kleene Closure?



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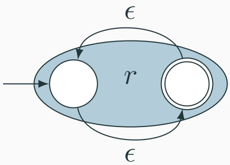
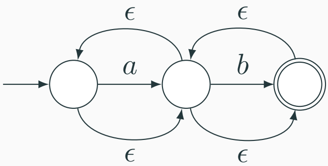


Now consider a^*b^* with this rule:



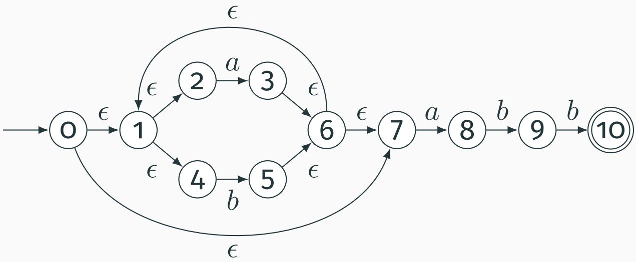
Is this right?

Translating REs into NFAs

Example: Translate $(a \mid b)^* abb$ into an NFA. Answer:

Translating REs into NFAs

Example: Translate $(a \mid b)^*abb$ into an NFA. Answer:



Show that the string "$aabb$" is accepted. Answer:

Translating REs into NFAs

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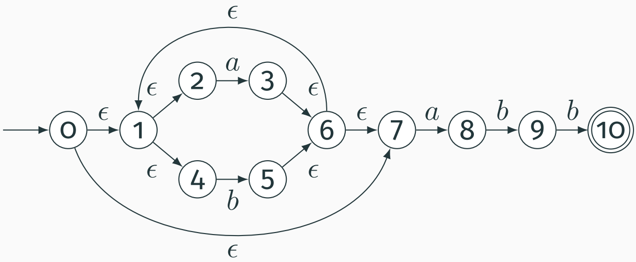

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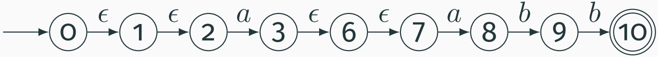

Simulating NFAs

Problem: you must follow the "right" arcs to show that a string is accepted. How do you know which arc is right?

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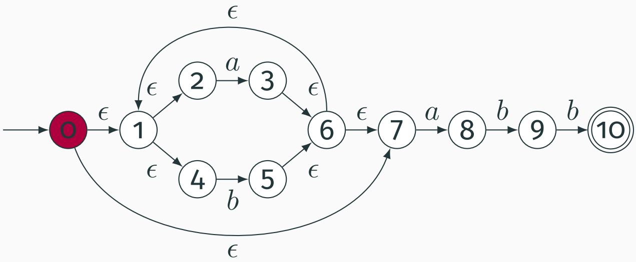
Solution: follow them all and sort it out later.

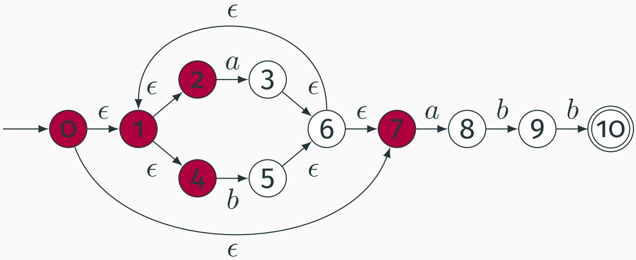
"Two-stack" NFA simulation algorithm:

1. Initial states: the ϵ-closure of the start state
2. For each character c,
   - New states: follow all transitions labeled c
   - Form the ϵ-closure of the current states
3. Accept if any final state is accepting

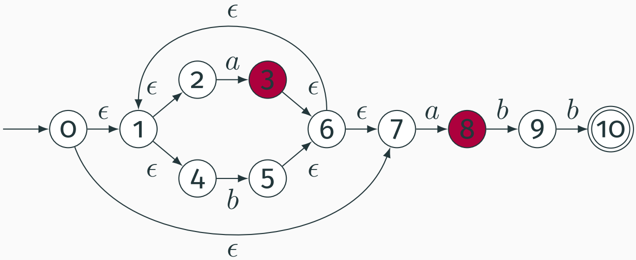
Simulating an NFA: $\cdot aabb$, Start



Simulating an NFA: $\cdot aabb$, ϵ-closure



Simulating an NFA: $a{\cdot}abb$



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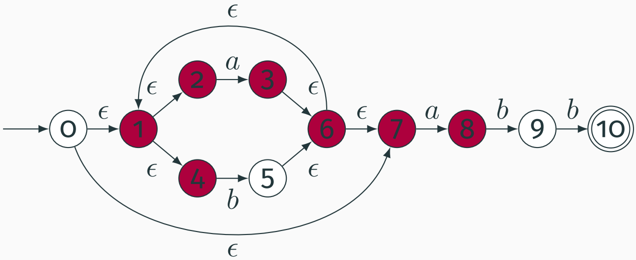
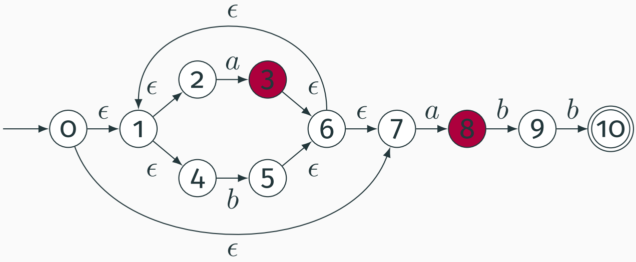


Simulating an NFA: $aa \cdot bb$



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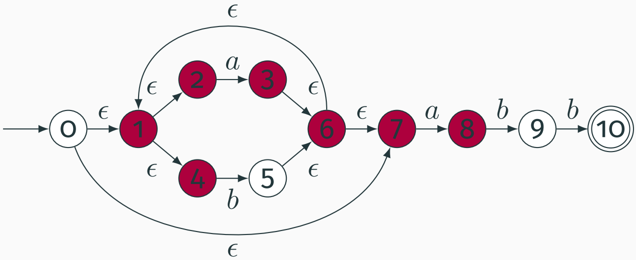
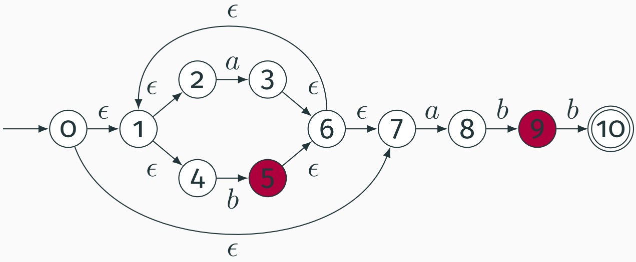


Simulating an NFA: $aab{\cdot}b$



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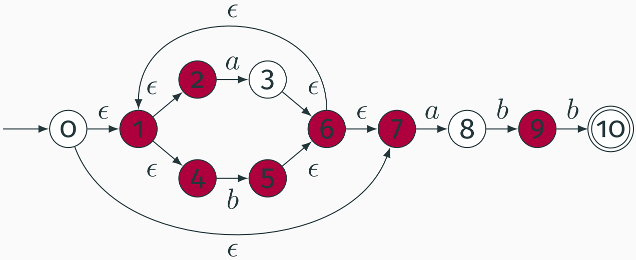
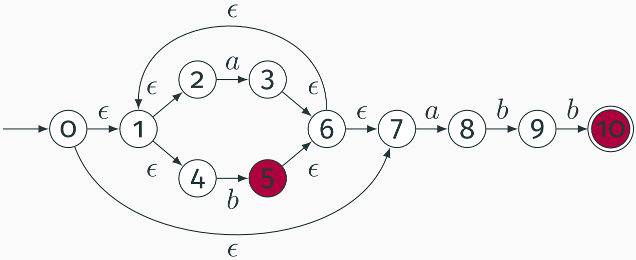
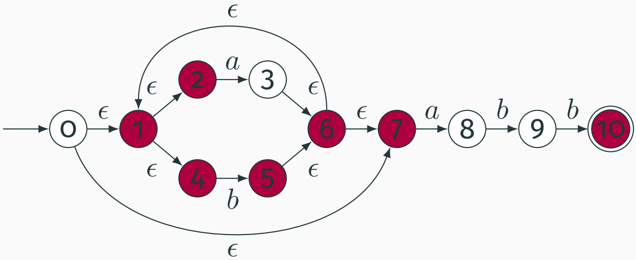


Simulating an NFA: $aabb\cdot$



Simulating an NFA: $aabb\cdot$, Done



Deterministic Finite Automata

Restricted form of NFAs:

- No state has a transition on ϵ
- For each state s and symbol a, there is at most one edge labeled a leaving s.

Very easy to check acceptance: simulate by maintaining current state. Accept if you end up on an accepting state. Reject if you end on a non-accepting state or if there is no transition from the current state for the next symbol.

Deterministic Finite Automata

```
{
   type token = ELSE | ELSEIF
}

rule token =
  parse "else"   { ELSE }
      | "elseif" { ELSEIF }
```

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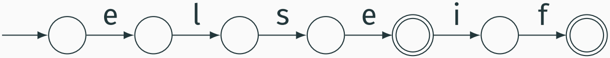
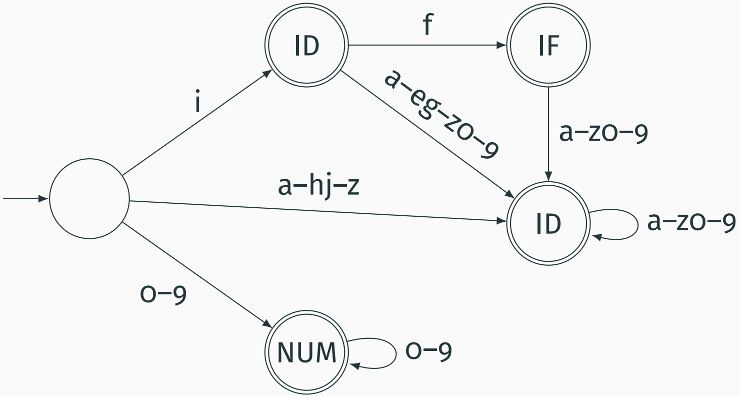


Deterministic Finite Automata

```
{ type token = IF | ID of string | NUM of string }

rule token =
  parse "if"                                     { IF }
      | ['a'-'z'] ['a'-'z' '0'-'9']* as lit { ID(lit) }
      | ['0'-'9']+                       as num { NUM(num) }
```



Building a DFA from an NFA

Subset construction algorithm

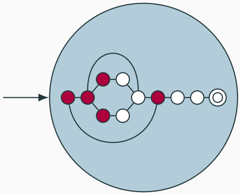
Simulate the NFA for all possible inputs and track the states that appear.

Each unique state during simulation becomes a state in the DFA.

The Subset Construction Algorithm

1. Create the start state of the DFA by taking the ε-closure of the start state of the NFA.
2. Perform the following for the new DFA state: For each possible input symbol:
   - Apply move to the newly-created state and the input symbol; this will return a set of states.
   - Apply the ε-closure to this set of states, possibly resulting in a new set. This set of NFA states will be a single state in the DFA.
3. Each time we generate a new DFA state, we must apply step 2 to it. The process is complete when applying step 2 does not yield any new states.
4. The finish states of the DFA are those which contain any of the finish states of the NFA.

Subset construction for $(a \mid b)^*abb$



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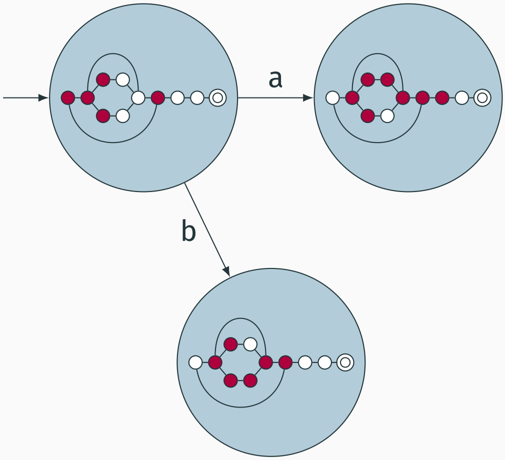

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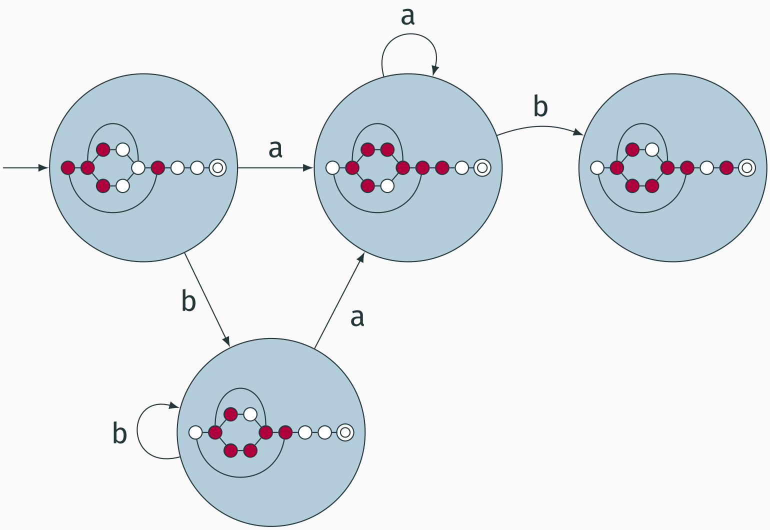
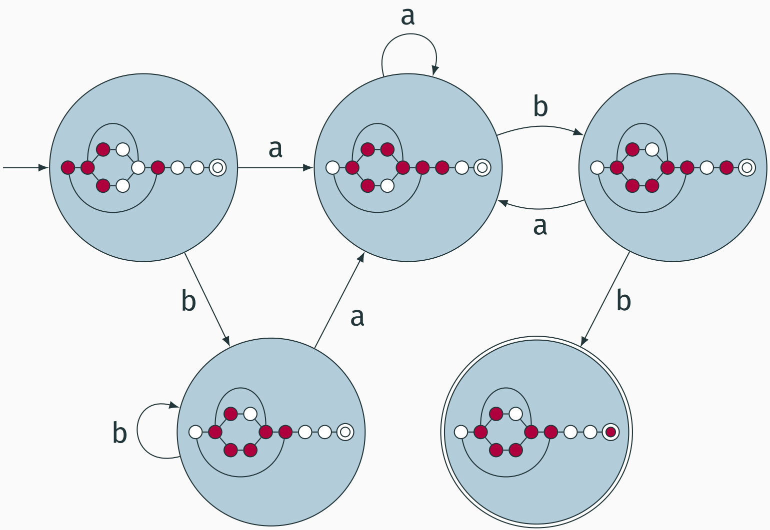
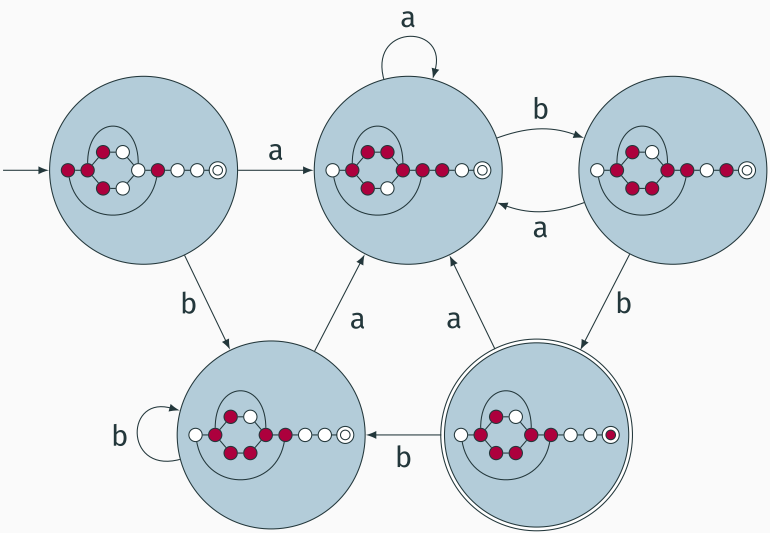


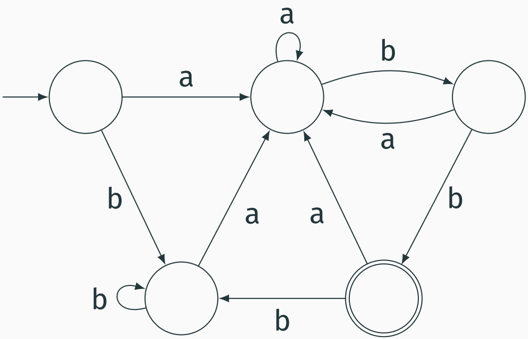
Subset construction for $(a \mid b)^* abb$



Subset construction for $(a \mid b)^*abb$

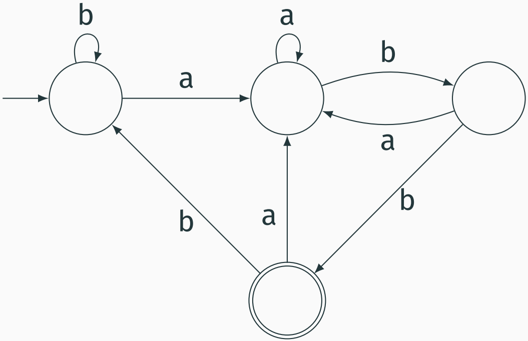


Result of subset construction for $(a \mid b)^*abb$



Is this minimal?

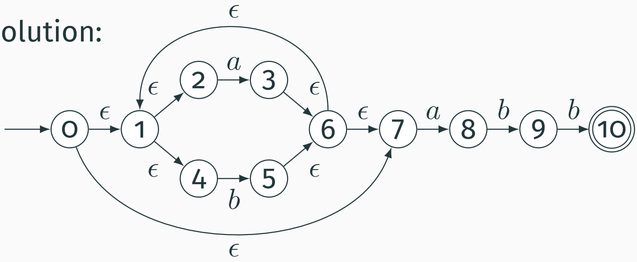
Minimized result for $(a \mid b)^*abb$



Transition Table Used In the Dragon Book

Problem: Translate $(a \mid b)^{*}abb$ into a DFA.

Solution:

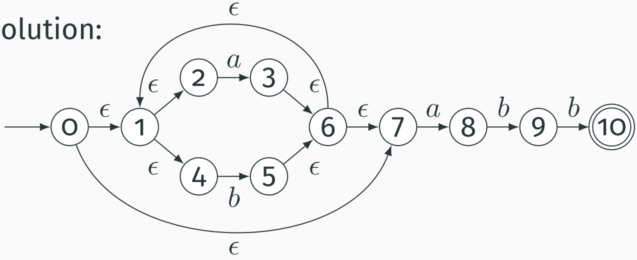


NFA State	DFA State	a	b

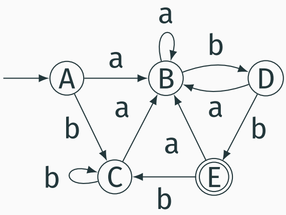
Transition Table Used In the Dragon Book

Problem: Translate $(a \mid b)^* abb$ into a DFA.

Solution:



NFA State	DFA State	a	b
{0,1,2,4,7}	A	B	C
{1,2,3,4,6,7,8}	B	B	D
{1,2,4,5,6,7}	C	B	C
{1,2,4,5,6,7,9}	D	B	E
{1,2,4,5,6,7,10}	E	B	C



Subset Construction

An DFA can be exponentially larger than the corresponding NFA.

n states versus 2^n

Tools often try to strike a balance between the two representations.

Lexical Analysis with Ocamllex

Constructing Scanners with Ocamllex

$$\boxed{\text{scanner.mll}} \xrightarrow[\text{(subset construction)}]{\text{ocamllex}} \boxed{\text{scanner.ml}}$$

An example:

scanner.mll

```
{ open Parser }

rule token =
  parse [' ' '\t' '\r' '\n'] { token lexbuf }
      | '+'                  { PLUS }
      | '-'                  { MINUS }
      | '*'                  { TIMES }
      | '/'                  { DIVIDE }
      | ['0'-'9']+ as lit    { LITERAL(int_of_string lit) }
      | eof                  { EOF }
```

Ocamllex Specifications

```
{
  (* Header: verbatim OCaml code; mandatory *)
}

(* Definitions: optional *)
let ident = regexp
let ...

(* Rules: mandatory *)
rule entrypoint1 [arg1 ... argn] =
  parse pattern1 { action (* OCaml code *) }
      | ...
      | patternn { action }
and entrypoint2 [arg1 ... argn]} =
  ...
and ...

{
  (* Trailer: verbatim OCaml code; optional *)
}
```

Patterns (In Order of Decreasing Precedence)

Pattern	Meaning
'c'	A single character
_	Any character (underline)
eof	The end-of-file
"foo"	A literal string
['1' '5' 'a'-'z']	“1,” “5,” or any lowercase letter
[^ '0'-'9']	Any character except a digit
(*pattern*)	Grouping
identifier	A pattern defined in the let section
pattern *	Zero or more *patterns*
pattern +	One or more *patterns*
pattern ?	Zero or one *patterns*
$pattern_1$ $pattern_2$	$pattern_1$ followed by $pattern_2$
$pattern_1 \mid pattern_2$	Either $pattern_1$ or $pattern_2$
pattern as *id*	Bind the matched pattern to variable *id*

An Example

```
{ type token = PLUS | IF | ID of string | NUM of int }
let letter = ['a'-'z' 'A'-'Z']
let digit = ['0'-'9']

rule token =
 parse [' ' '\n' '\t'] { token lexbuf } (* Ignore whitespace *)

     | '+' { PLUS }                     (* A symbol *)

     | "if" { IF }                      (* A keyword *)
                                        (* Identifiers *)
     | letter (letter | digit | '_')* as id { ID(id) }
                                        (* Numeric literals *)
     | digit+ as lit { NUM(int_of_string lit) }

     | "/*" { comment lexbuf }          (* C-style comments *)

and comment =
  parse "*/" { token lexbuf } (* Return to normal scanning *)
      | _ { comment lexbuf }  (* Ignore other characters *)
```

Nested Comments

```
{ type token = PLUS | ID of string | NUM of int }

let letter = ['a'-'z' 'A'-'Z']
let digit = ['0'-'9']

rule token =
 parse [' ' '\n' '\t'] { token lexbuf } (* Ignore whitespace *)

     | '+' { PLUS }                     (* A symbol *)

     | letter (letter | digit | '_')* as id { ID(id) }
     | digit+ as lit { NUM(int_of_string lit) }

     | "/*" { comment 0 lexbuf }          (* C-style comments *)

and comment level =
  parse "*/" { if level == 0 then token lexbuf
        else comments (level - 1) lexbuf }
      | "/*" { comment (level + 1) lexbuf }
      | _ { comment level lexbuf }  (* Ignore other characters *)
```

Free-Format Languages

Typical style arising from scanner/parser division

Program text is a series of tokens possibly separated by whitespace and comments, which are both ignored.

- keywords (if while)
- punctuation (, (+)
- identifiers (foo bar)
- numbers (10 -3.14159e+32)
- strings ("A String")

Free-Format Languages

Java C C++ C# Algol Pascal

Some deviate a little (e.g., C and C++ have a separate preprocessor)

But not all languages are free-format.

Python

The Python scripting language groups with indentation

```
i = 0
while i < 10:
    i = i + 1
    print i     # Prints 1, 2, ..., 10

i = 0
while i < 10:
    i = i + 1
print i         # Just prints 10
```

This is succinct, but can be error-prone.

How do you wrap a conditional around instructions?

Syntax and Language Design

Does syntax matter? Yes and no

More important is a language's *semantics*—its meaning.

The syntax is aesthetic, but can be a religious issue.

But aesthetics matter to people, and can be critical.

Verbosity does matter: smaller is usually better.

Too small can be problematic: APL is a succinct language with its own character set.

There are no APL programs, only puzzles.

Syntax and Language Design

Some syntax is error-prone. Classic FORTRAN example:

```
DO 5 I = 1,25   ! Loop header (for i = 1 to 25)
DO 5 I = 1.25   ! Assignment to variable DO5I
```

Trying too hard to reuse existing syntax in C++:

```
vector< vector<int> > foo;
vector<vector<int>> foo; // Syntax error
```

C distinguishes $>$ and $>>$ as different operators.

Bjarne Stroustrup tells me they have finally fixed this.